Slow light using spin coherence and V-type electromagnetically induced transparency in [110] strained quantum wells

Shu-Wei Chang and Shun Lien Chuang
Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Illinois 61801, USA

Connie J. Chang-Hasnain
Department of Electrical Engineering and Computer Sciences, University of California at Berkeley, Berkeley, California 94720, USA

Hailin Wang
Department of Physics and Oregon Center for Optics, University of Oregon, Eugene, Oregon 97403, USA

Received September 14, 2006; revised November 22, 2006; accepted November 28, 2006; posted December 8, 2006 (Doc. ID 75050); published March 15, 2007

We model the slow-light phenomenon using electromagnetically induced transparency from spin coherence in [110] quantum wells. The long electron-spin lifetime in [110] quantum wells and the strain-induced shift of the light-hole-like excitonic transition energy below those of the heavy-hole-like continuum states can enhance the performance of slow light. The optical anisotropy due to band mixing and strain in [110] quantum wells is properly taken into consideration. By using light-hole-like excitons, the delay of a probe light with TM polarization in an optical waveguide controllable by an intense pump is made possible with the aid of long spin coherence. © 2007 Optical Society of America

1. INTRODUCTION

Using semiconductor quantum wells (QWs) as a potential slow-light device has been experimentally and theoretically verified by using population oscillation (PO). In addition to PO, a steep dispersion due to V-type electromagnetically induced transparency (EIT) from the spin coherence has been proposed by using light-hole (LH) excitons in QWs grown along the [001] crystal axis. Both V-type EIT and PO are coherent effects in semiconductor QWs and are potential candidates to implement semiconductor-based slow-light devices. However, for the LH transition in [001] QW, absorption from the heavy-hole (HH) continuum and rapid relaxation of holes from LH bands to HH bands greatly complicate the experimental demonstration and limit potential applications.

We propose to use strain effect on a [110] QW structure to lift the LH-like subband above the HH subband for the V-type EIT in QWs. In this way, the LH-like exciton will not lie in the HH continuum, and the absorption as well as relaxation from the HH continuum will be absent. The strain can be achieved using a strained QW due to lattice difference or external stress. In addition, electron-spin lifetime is also expected to be much longer in [110] QWs than that in [001] QWs, due to the absence of the Dyakonov–Perel mechanism. The longer spin relaxation time in QWs grown along the [110] direction at room temperature has been predicted and experimentally verified. The long electron-spin lifetime makes it possible to create steep dispersion via LH excitons using V-type EIT.

There are intrinsic differences between [001] and [110] QWs. Unlike [001] QWs, valence band mixing takes place even at the Brillouin zone (BZ) center in [110] QWs. In this case, pure HH or LH characteristics cannot be assigned to QW excitons, though they can still be roughly called HH-like or LH-like, depending on their compositional wave functions. The optical selection rules of QW excitons are characterized by the band-edge states in the conduction and valence bands. At the BZ center of [110] QWs, the HH-3/2,−3/2 and LH-3/2,1/2 components couple to each other and form one state (|φ_{hh1}⟩), while the HH-3/2,3/2 and LH-3/2,−1/2 from the other (|φ_{hh2}⟩). The conduction states can still be characterized by spins (|φ_{c1}⟩ and |φ_{c2}⟩). The mixing of the HH and LH components modifies the relative weights of the components |X⟩ (along [001]) and |Y⟩ (along [110]) of the states at the BZ center, and changes the original selection rule of the circularly polarized light in [001] QWs into the elliptically polarized light in [110] QWs. As shown in Fig. 1(a), for the transverse electric-polarized (TE-polarized) pump, well-defined transitions can be induced from state |φ_{hh1}⟩ to |φ_{c1}⟩ and from state |φ_{hh2}⟩ to |φ_{c1}⟩ by left-hand or right-hand elliptically polarized optical fields (\tilde{u}_1 and \tilde{u}_2). This
polarizations will rotate periodically as indicated by the arrows.

When the direction of the external stress is applied along the growth direction, the TM-polarized signal will only experience this steep dispersion, the group velocity of the TM signal can be greatly reduced.

There are two ways to use QW strain to make the highest valence bands LH-like. The first one uses lattice-mismatched materials. The internal stress due to different lattice constants between the substrate and the QW material can lift up the LH-like bands to the top of HH-like bands. An example is the ternary component In$_{1-x}$Ga$_x$As grown on the substrate InP used in optical communication. For In$_{1-x}$Ga$_x$As grown on InP, the LH-like state energy level will be above that of the HH-like state energy when the compositional fraction of gallium $x$ is larger than 0.468, due to the biaxial tensile strain. The second approach is to apply an external uniaxial stress to artificially generate strain in QWs. This approach is mainly focused on the lattice-matched QW system, e.g., GaAs/Al$_{1-x}$Ga$_x$As. The external stress will induce a strain distribution in the whole system. If the direction and the magnitude of the applied external stress are proper, it is also possible to lift the energy of the LH band edge to the top of the HH band edge. To make the highest valence band LH-like for [001] or [110] QWs, the external uniaxial stress has to be applied along the growth direction. Figure 2 shows the pump-signal configuration for this case. Since the external stress is applied from the top and the bottom of the QW by the mechanical appliance, the TE pump and TM signal will be incident from the edge of the QW using waveguide geometry.

Fig. 1. V-type EIT using LH excitons in [110] QWs. (a) Selection rule. The black line indicates the polarization induced by the TE pump, while the solid and dashed gray lines mean the respective transitions induced by the TM-polarized signal. (b) Pump–probe configuration. The optical field of the pump can be decomposed into two elliptically polarized components ($\hat{u}_1$ and $\hat{u}_2$). The two polarizations will rotate periodically as indicated by the arrows.

is analogous to the well-defined transitions induced by circularly polarized lights in [001] QWs. For transverse magnetic-polarized (TM-polarized or ẑ-polarized) signals, both transitions from state $|\hat{\phi}_h\rangle$ to $|\phi_\uparrow\rangle$ and from state $|\hat{\phi}_h\rangle$ to $|\phi_\downarrow\rangle$ can be induced. Figure 1(b) shows the pump-signal configuration of V-type EIT. The pump is incident along the ẑ direction. It can be decomposed into two left- and right-hand elliptically polarized components with polarizations $\hat{u}_1$ and $\hat{u}_2$. The coherences between states $|\hat{\phi}_h\rangle$ and $|\phi_\uparrow\rangle$ and between states $|\hat{\phi}_h\rangle$ and $|\phi_\downarrow\rangle$ can thus be generated by the pump. The spin coherences between states $|\phi_\uparrow\rangle$ and $|\phi_\downarrow\rangle$ and between states $|\hat{\phi}_h\rangle$ and $|\hat{\phi}_h\rangle$ then give rise to the absorption dip and the steep dispersion for the induced transitions of the signal from state $|\hat{\phi}_h\rangle$ to $|\phi_\uparrow\rangle$ and from state $|\hat{\phi}_h\rangle$ to $|\phi_\downarrow\rangle$. With this steep dispersion, the group velocity of the TM signal can be greatly reduced.

From the above analysis, the LH characteristic of the valence bands involved in the V-type EIT is important because the TM-polarized signal will only experience this steep dispersion if the dipole moment along the ẑ direction is nonzero. In typical unstrained [001] or [110] QWs, the two highest valence bands are well approximated as purely HH or HH-like with spin up or down. Usually, it is the next two valence bands that show significant LH characteristics, and they can be used. However, the corresponding transitions of the LH-like exciton energy lie in the absorption spectrum of the HH-like continuum. Although it is possible to induce the quantum coherence of the continuum states, HH-like continuum states have insignificant dipole moments along the growth direction and usually play the role of unwanted dissipation for both the pump and signal. Also, small but finite coupling between LH and HH components brings about undesired optical dephasing, which can be understood as Fano-type coupling between a discrete state (LH-like exciton) and continuum states (HH-like continuum). It is necessary to lift the LH-like states to the top of the HH-like states or to mix significant portions of the LH components to the original HH-like states at the band edge. The strain in a QW can change the effective band offset and coupling between the HH and LH components. With the change of the effective band offset and coupling between HH and LH components, the nature of the states involved in the V-type EIT can be more flexibly engineered to serve the purpose of slow light.

Fig. 2. Pump–probe configuration when a uniaxial stress is applied to a GaAs/Al$_{1-x}$Ga$_x$As [110] QW. The uniaxial stress is applied along the z axis. The pump and signal are incident onto the waveguide from the edge of the QW.
As shown in extensive earlier studies, coherent nonlinear optical responses in semiconductors are strongly modified by underlying many-body Coulomb interactions. More recent experimental studies have also revealed that nonlinear optical processes associated with electron–spin coherences depend crucially on the underlying many-body Coulomb interactions. While a complete theoretical description of EIT processes in semiconductors will require the inclusion of these many-body effects, a treatment of EIT arising from electron–spin coherence beyond a phenomenological model still remains a considerable theoretical challenge. The optical selection rules discussed in this work, which are essential to the proposed EIT process, however, remain valid in spite of the underlying many-body effects. We hope that our work will stimulate further theoretical efforts to tackle the more difficult theoretical problem of the effects of many-body interactions.

In this paper, we will show the selection rules essential to V-type EIT for [110] QWs. In particular, the effect of the strain on the selection rules will be included. The density-matrix formalism will be used to calculate the signal and pump polarizations as well as the corresponding absorption and real part of the refractive index. We will focus on the AlGaAs/GaAs system under external stress. The formulation of the lattice-mismatch system is similar. Finally, we will show achievable slowdown factors for V-type EIT using [110] strained QWs.

2. THEORETICAL MODEL

For [110] QWs at the BZ center, the Luttinger–Kohn (LK) Hamiltonian, including the strain effect for the “J=3/2” valence bands, can be written as

\[ H_{\text{LK}}^{[110]}(k_x - 0, k_y = 0, k_z = -i \partial_z) = V_e(z) I_4 + H'_e^{[110]}(z) \]

\[ H'_e^{[110]}(z) = -P^e I_4 - \left[ \begin{array}{cccc}
Q^e & -S^e & R^e & 0 \\
S^e & -Q^e & 0 & R^e \\
R^e & 0 & -Q^e & S^e \\
0 & R^e & S^e & Q^e 
\end{array} \right] \]

\[ P^e = -\alpha_i (e_{xx} + e_{yy} + e_{zz}), \]

\[ Q^e = -\frac{b}{2} e_{xx} + \frac{1}{2} \left( \frac{b}{2} - \frac{\sqrt{3}d}{2} \right) e_{yy} + \frac{1}{2} \left( \frac{\sqrt{3}d}{2} \right) e_{zz}, \]

\[ R^e = -\frac{\sqrt{3}b}{2} e_{xx} - \frac{1}{2} \left( \frac{\sqrt{3}b}{2} + \frac{d}{2} \right) e_{yy} - \frac{1}{2} \left( \frac{\sqrt{3}b}{2} - \frac{d}{2} \right) e_{zz} - i d e_{xy}, \]

where \( k_x, k_y, \) and \( k_z \) are the three wave vectors propagating along the three crystal axes [001], [110], and [110]; \( V_e(z) \) is the effective potential experienced by valence electrons at the BZ center; \( H'_e^{[110]}(z) \) describes the strain effect for [110] orientation; \( P^e, Q^e, R^e, \) and \( S^e \) are the strain matrix elements; \( a_i, b, \) and \( d \) are three deformation potentials in the strain Hamiltonian from Bir and Pikus; \( \alpha_i, \beta_i, \) and \( \mu_i \) are parameters that describe the differential operator, with \( \mu_i \) being responsible for the coupling between different hole components at the BZ center for the [110] QW; and \( \gamma_1, \gamma_2, \) and \( \gamma_3 \) are three Luttinger’s parameters. In Eq. (1), various parameters take the respective bulk values in the QW and barrier regions and are therefore position dependent.

If the diagonal strain components \( e_{xx}, e_{yy}, \) and \( e_{zz} \) are nonzero, it is possible to make the highest valence subbands LH-like. We can write the corresponding time-independent LH-like exciton wave functions \( \psi_{\text{ex}}^{\mu e}(r) (i = 1, 2) \) in variational form as

\[ \psi_{\text{ex}}^{\mu e}(r) = \varphi_{\text{ex}}(\rho_e - \rho_h) \phi_{\text{ex}}(z_e) |S\sigma\rangle \otimes \text{TR}[\bar{\phi}(z_h)], \]

where \( \varphi_{\text{ex}}(\rho_e - \rho_h) \) is the in-plane variational function; \( \phi_{\text{ex}}(z_e) |S\sigma\rangle \) is the lowest quantized state with spin \( \sigma \) and isotropic Bloch state \( |S\rangle \) in the conduction band; \( \bar{\phi}(z_h) \) \((i=1, 2)\) is the quantized four-component spinor in the four-band model in the electron representation, and TR means time-reversal operation, which takes the complex conjugate of the spinor wave function and makes the time evolution of the total wave function an exponential term oscillating with the interband energy. From Eq. (1), if the tensor components \( e_{xx} \) and \( e_{yz} \) are absent, the \([3/2, 3/2]\) component will only couple to the \([3/2, -1/2]\) component, while the \([3/2, -3/2]\) component will only couple to the \([3/2, 1/2]\) component at the BZ center for [110] QWs. The two LH-like spinors satisfy the eigenvalue problem specified by the Hamiltonian operator in Eq. (1). Up to a phase vector, their expressions can be written as
\[ \tilde{\phi}_{h1}(z_h) = \Psi(z_h) | \frac{3}{2}, -\frac{3}{2} \rangle + \Phi(z_h) | \frac{1}{2}, \frac{1}{2} \rangle, \]
\[ \tilde{\phi}_{h2}(z_h) = \Psi(z_h) | \frac{3}{2}, -\frac{3}{2} \rangle + \Phi(z_h) | \frac{3}{2}, -\frac{1}{2} \rangle, \]
\[ 1 = \int_{-\infty}^{\infty} dz_h (|\Psi(z_h)|^2 + |\Phi(z_h)|^2), \quad (3) \]

where \( \Psi(z_h) \) and \( \Phi(z_h) \) are the two quantized wave functions to be solved numerically with the corresponding normalization condition shown in Eq. (3). The effective dipole moments between electron and hole components for various excitons can be written as
\[ \langle \phi_e | e^r | \tilde{\phi}_{h1} \rangle = \langle \phi_e | \Psi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{3}{2} \rangle + \langle \phi_e | \Phi \rangle \langle S^e | e^r | \frac{1}{2}, \frac{1}{2} \rangle, \]
\[ \langle \phi_e | e^r | \tilde{\phi}_{h2} \rangle = \langle \phi_e | \Psi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{3}{2} \rangle + \langle \phi_e | \Phi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{1}{2} \rangle, \]
\[ \langle \phi_e | e^r | \tilde{\phi}_{h1} \rangle = \langle \phi_e | \Psi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{3}{2} \rangle + \langle \phi_e | \Phi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{3}{2} \rangle, \]
\[ \langle \phi_e | e^r | \tilde{\phi}_{h2} \rangle = \langle \phi_e | \Psi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{3}{2} \rangle + \langle \phi_e | \Phi \rangle \langle S^e | e^r | \frac{3}{2}, -\frac{1}{2} \rangle. \quad (4) \]

Denote the overlap integrals \( \langle \phi_e | \Psi \rangle \) and \( \langle \phi_e | \Phi \rangle \) as \( \kappa \) and \( \eta \), respectively. Since the eigenvalue problem of the Hamiltonian operator in Eq. (1) is a real differential equation, the phases of the overlap integrals \( \kappa \) and \( \eta \) will differ only by 0 or \( \pi \). Using the expressions of the various dipole moments between various Bloch states,
\[ \langle S^\uparrow | e^r | \frac{3}{2}, \frac{3}{2} \rangle = -r_{ch} \hat{e}_z, \]
\[ \langle S^\uparrow | e^r | \frac{1}{2}, \frac{1}{2} \rangle = \sqrt{\frac{2}{3}} r_{ch} \hat{e}_z, \]
\[ \langle S^\uparrow | e^r | \frac{1}{2}, -\frac{1}{2} \rangle = \frac{1}{\sqrt{3}} r_{ch} \hat{e}_z, \]
\[ \langle S^\uparrow | e^r | \frac{3}{2}, -\frac{3}{2} \rangle = 0, \]
\[ \langle S^\uparrow | e^r | \frac{3}{2}, \frac{3}{2} \rangle = 0, \]
\[ \langle S^\uparrow | e^r | \frac{1}{2}, \frac{1}{2} \rangle = -\frac{1}{\sqrt{3}} r_{ch} \hat{e}_z, \]
\[ \langle S^\uparrow | e^r | \frac{1}{2}, -\frac{1}{2} \rangle = \sqrt{\frac{2}{3}} r_{ch} \hat{e}_z, \]
\[ \langle S^\uparrow | e^r | \frac{3}{2}, -\frac{3}{2} \rangle = r_{ch} \hat{e}_z, \]
\[ \hat{e}_z = \frac{1}{\sqrt{2}} (\hat{x} \pm i \hat{y}). \quad (5) \]

We can write Eq. (4) as
\[ \langle \phi_e | e^r | \tilde{\phi}_{h1} \rangle = e_{ch} \eta \sqrt{\frac{2}{3}} \hat{z}, \]
\[ \langle \phi_e | e^r | \tilde{\phi}_{h2} \rangle = e_{ch} \left( -\kappa \hat{e}_z + \frac{\eta}{\sqrt{3}} \hat{e}_z \right) \]
\[ = e_{ch} \left( -\frac{\kappa}{\sqrt{2}} - \frac{\eta}{\sqrt{6}} \right) \hat{x} - i \left( \frac{\kappa}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \right) \hat{y}, \]
\[ \langle \phi_e | e^r | \tilde{\phi}_{h1} \rangle = e_{ch} \left( \kappa \hat{e}_z - \frac{\eta}{\sqrt{3}} \hat{e}_z \right) \]
\[ = e_{ch} \left( \frac{\kappa}{\sqrt{2}} - \frac{\eta}{\sqrt{6}} \right) \hat{x} - i \left( \frac{\kappa}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \right) \hat{y}, \]
\[ \langle \phi_e | e^r | \tilde{\phi}_{h2} \rangle = e_{ch} \eta \sqrt{\frac{2}{3}} \hat{z}. \quad (6) \]

The optical anisotropy due to band mixing and strain have been extensively discussed in the literature, e.g., see Ref. 13. Our goal is to find the counterparts of the corresponding in-plane circular-polarization selection rules for the exciton in the [001] QW, which make the successive calculation proceed clearly. To incorporate the selection rule of this LH-like system correctly, we seek two in-plane and complex unit vectors \( \hat{u}_1 \) and \( \hat{u}_2 \), such that they are orthogonal to the two dipole moments \( \langle \phi_e | e^r | \tilde{\phi}_{h2} \rangle \) and \( \langle \phi_e | e^r | \tilde{\phi}_{h1} \rangle \),
\[ \hat{u}_1 \cdot \langle \phi_e | e^r | \tilde{\phi}_{h2} \rangle = 0, \]
\[ \hat{u}_2 \cdot \langle \phi_e | e^r | \tilde{\phi}_{h1} \rangle = 0. \quad (7) \]

Thus, the correlation between states \( \tilde{\phi}_{h2} \) and \( \phi_e \) is decoupled from the field with polarization vector \( \hat{u}_1 \) and will be induced only by the component with polarization vector \( \hat{u}_2 \). Similarly, the correlation between states \( \tilde{\phi}_{h1} \) and \( \phi_e \) is induced only by the component with polarization vector \( \hat{u}_1 \). The expressions of vectors \( \hat{u}_1 \) and \( \hat{u}_2 \) are written in terms of circularly polarized vectors \( \hat{e}_z \) as
\[ \hat{u}_1 = \frac{\kappa}{\sqrt{\kappa^2 + \eta^2 / 3}} \hat{e}_z + \frac{\eta \sqrt{3}}{\sqrt{\kappa^2 + \eta^2 / 3}} \hat{e}_x, \]
\[ \hat{u}_2 = \frac{\eta \sqrt{3}}{\sqrt{\kappa^2 + \eta^2 / 3}} \hat{e}_z + \frac{\kappa}{\sqrt{\kappa^2 + \eta^2 / 3}} \hat{e}_x. \quad (8) \]

The two vectors \( \hat{u}_1 \) and \( \hat{u}_2 \) describe elliptically polarized lights if the propagation direction is along the growth direction. An important fact is that the two vectors \( \hat{u}_1 \) and \( \hat{u}_2 \) are, in general, nonorthogonal. Orthogonality only holds when either \( \kappa = 0 \) or \( \eta = 0 \). Although these two complex unit vectors are not necessarily orthogonal, we can still use them to expand the TE electric field as long as they are linearly independent. The total electric field is composed of positive and negative frequency components and can be expressed as
\[ \mathbf{E}(t) = \frac{1}{2}[\mathcal{E}(t) + \mathcal{E}^*(t)], \] (9)

where \( \mathcal{E}(t) \) is the positive-frequency part, while \( \mathcal{E}^*(t) \) is the negative frequency part. The positive frequency part \( \mathcal{E}(t) \) contains a pump component and a signal component:

\[ \mathcal{E}(t) = \mathbf{E}_pe^{-i\omega_st} + \mathbf{E}_se^{-i\omega_st}, \] (10)

where \( \mathbf{E}_p \) and \( \mathbf{E}_s \) are the amplitudes of the pump and signal, respectively. The pump is a linearly polarized light propagating along the growth direction. In terms of vectors \( \vec{u}_1 \) and \( \vec{u}_2 \), its amplitude is written as

\[ \mathbf{E}_p = E_{p,1}\vec{u}_1 + E_{p,2}\vec{u}_2 = \frac{E_p\sqrt{|\kappa|^2 + |\eta|^2/3}}{\sqrt{2}} \left( e^{-i\phi_K} - e^{i\phi_p} \right) \vec{u}_1 \]
\[ + \frac{E_p\sqrt{|\kappa|^2 + |\eta|^2/3}}{\sqrt{2}} \left( e^{i\phi_K} - e^{-i\phi_p} \right) \vec{u}_2, \] (11)

where \( E_p \) is the magnitude of the pump, and \( \phi_p \) is the azimuthal angle of the pump field measured from the x-axis. The Rabi frequencies due to the intense pump can be written as

\[ \Omega_{p,12} = \frac{E_p}{2\hbar}\left(\phi_p |\vec{e}| \Omega_{\vec{h}2} + \frac{E_{p,1}\vec{u}_1}{2\hbar} \cdot (\phi_p |\vec{e}| \Omega_{\vec{h}2}) \right) \]
\[ = \frac{er_{ch}E_p}{2\hbar} \left( e^{-i\phi_p}(\sqrt{3} - e^{-i\phi_P}) \right), \]
\[ \Omega_{p,11} = \frac{E_p}{2\hbar}(\phi_p |\vec{e}| \Omega_{\vec{h}1} + \frac{E_{p,1}\vec{u}_1}{2\hbar} \cdot (\phi_p |\vec{e}| \Omega_{\vec{h}1}) \]
\[ = \frac{er_{ch}E_p}{2\hbar} \left[ e^{-i\phi_P}K - e^{i\phi_p}(\sqrt{3}) \right]. \] (12)

The signal is a weak beam propagating along the QW plane with a TM-polarized signal:

\[ \mathbf{E}_s = E_{s,z}\hat{z}. \] (13)

The corresponding Rabi frequency can be written as

\[ \Omega_{s,11} = \frac{E_s}{2\hbar}(\phi_s |\vec{e}| \Omega_{\vec{h}1} + \frac{E_{s,1}\vec{u}_1}{2\hbar} \cdot (\phi_s |\vec{e}| \Omega_{\vec{h}1}) \]
\[ = \frac{er_{ch}E_s}{2\hbar} \left( e^{-i\phi_s}(\sqrt{3}) - e^{i\phi_s}(\sqrt{3}) \right), \]
\[ \Omega_{s,22} = \frac{E_s}{2\hbar}(\phi_s |\vec{e}| \Omega_{\vec{h}2} + \frac{E_{s,2}\vec{u}_1}{2\hbar} \cdot (\phi_s |\vec{e}| \Omega_{\vec{h}2}) \]
\[ = \frac{er_{ch}E_s}{2\hbar} \left( e^{-i\phi_s}(\sqrt{3}) - e^{i\phi_s}(\sqrt{3}) \right). \] (14)

Instead of working on the detailed dynamics of excitons with different angular momenta,\(^4\) we focus only on the occupations and polarizations of each level.\(^5\) In our model, the population exchange of excitons with different angular momenta is effectively described by the spin-flipping terms in our equations. From Refs. 2 and 15, the equations of motion of various effective populations and polarizations can be written as

\[ \partial_t N_1 = -\gamma(N_1 - N_1^{(0)}) - \gamma_s(N_1 - N_s) + i[\Omega_{s,1}(t)P_{11} - P_{11}\Omega_{s,1}(t)], \]
\[ \partial_t N_2 = -\gamma(N_2 - N_2^{(0)}) - \gamma_s(N_2 - N_s) + i[\Omega_{s,2}(t)P_{22} - P_{22}\Omega_{s,2}(t)], \]
\[ \partial_t N_s = -\gamma_s(N_s - N_s^{(0)}) + i[\Omega_{s,1}(t)P_{11} - P_{11}\Omega_{s,1}(t)], \]
\[ \partial_t N_{12} = \gamma_s(N_1 - N_1^{(0)}) + \gamma_s(N_2 - N_2^{(0)}) - \gamma_s(N_1 - N_2) + i[\Omega_{s,1}(t)P_{12} - P_{12}\Omega_{s,1}(t)], \]
\[ \partial_t N_{21} = \gamma_s(N_2 - N_2^{(0)}) + \gamma_s(N_1 - N_1^{(0)}) - \gamma_s(N_1 - N_2) + i[\Omega_{s,2}(t)P_{21} - P_{21}\Omega_{s,2}(t)], \]
\[ \partial_t n_1 = -i[\omega_{ix} - i\Gamma_2(N_1 - N_2,N_1 - N_1)]P_{11} - i\Omega_{s,1}(t)(N_1 - N_1) \]
\[ - i\Omega_{s,1}(t)P_{11} + iP_{11}\Omega_{s,1}(t), \]
\[ \partial_t n_2 = -i[\omega_{ix} - i\Gamma_2(N_1 - N_2,N_1 - N_1)]P_{12} - i\Omega_{s,2}(t)(N_1 - N_2) \]
\[ - i\Omega_{s,2}(t)P_{12} + iP_{12}\Omega_{s,2}(t), \]
\[ \partial_t n_{12} = -i[\omega_{ix} - i\Gamma_2(N_1 - N_2,N_1 - N_1)]P_{21} - i\Omega_{s,1}(t)(N_1 - N_2) \]
\[ - i\Omega_{s,1}(t)P_{21} + iP_{21}\Omega_{s,1}(t), \]
\[ \partial_t n_{21} = -i[\omega_{ix} - i\Gamma_2(N_1 - N_2,N_1 - N_1)]P_{22} - i\Omega_{s,2}(t)(N_1 - N_2) \]
\[ - i\Omega_{s,2}(t)P_{22} + iP_{22}\Omega_{s,2}(t), \]
\[ \partial_t N_{12} = -\Gamma_2 P_{12} + i[\Omega_{s,1}(t)P_{12} - P_{12}\Omega_{s,1}(t)] + i[\Omega_{s,2}(t)P_{21} - P_{21}\Omega_{s,2}(t)] - i[\Omega_{s,1}(t)P_{12} - P_{12}\Omega_{s,1}(t)], \] (15)

where \( N_1 \) and \( N_2 \) are the effective populations of the spin-up and spin-down components in the conduction bands; \( N_{12} \) and \( N_{21} \) are the effective populations in the valence bands 1 and 2; \( P_{11}, P_{12}, P_{21}, P_{22} \), and \( P_{12} \) are the polarizations of the corresponding state labels; the populations \( N_1^{(0)} = N_2^{(0)} = 0 \) and \( N_1^{(0)} = N_2^{(0)} = 1 \) are the corresponding counterparts in absence of the pump; \( \gamma_s \), \( \gamma_s \), and \( \gamma \) ( \( = \gamma_s + \gamma_\perp \) ) are the spontaneous emission constant of a z-polarized photon, the spontaneous emission constant of an in-plane polarized photon, and the total spontaneous constant, respectively; \( \gamma_s \) is the spin-flip constant in the conduction band; \( \Gamma_\perp \) is the transition constant between states 1 and 2; \( \omega_{ix} \) is the exciton frequency; \( \Gamma_2(N_1 - N_2,N_1 - N_1) \) is the dephasing constant, which includes the effect of excitation-induced dephasing (EID); and \( \gamma_s \) and \( \Gamma_\perp \) are the spin-dephasing constants in the conduction and valence bands. Due to the created electron-hole pairs, the spin relaxation and dephasing times can decrease via the Bir–Arnon–Pikus (BAP) mechanism as the pump intensity increases.\(^6\) However, it suffices to keep spin relaxation and dephasing times constant for a first demonstration. Also, the generation of the electron-hole pairs leads to the screening and energy renormalization from the many-body effect. Since we are only inter-
ested in the optical response within a narrow frequency range, and the screening and renormalization are usually global changes in various optical spectra, we will omit these higher-order effects, while maintaining the clarity of the model.

We first find the bias solution determined by the pump. After setting signal Rabi frequencies to zero, the dynamics equations due to the pump are written as

\[
\begin{align*}
\dot{N}_1^{(p)} &= -\gamma(N_1^{(p)} - N_1^{(0)}) - \gamma_s(N_0^{(p)} - N_1^{(p)}) \\
&\quad + \frac{i}{2} \Omega_{p,1} e^{-i\omega_p t} P_{1}^{(p)} - \frac{1}{2} \Omega_{p,2} e^{i\omega_p t}, \\
\dot{N}_2^{(p)} &= -\gamma(N_2^{(p)} - N_2^{(0)}) - \gamma_s(N_0^{(p)} - N_2^{(p)}) \\
&\quad + \frac{i}{2} \Omega_{p,1} e^{i\omega_p t} P_{2}^{(p)} - \frac{1}{2} \Omega_{p,2} e^{-i\omega_p t}, \\
\dot{N}_1^{(p)} &= \gamma_s(N_0^{(p)} - N_1^{(0)}) + \gamma_s(N_2^{(p)} - N_1^{(p)}) - \Gamma_s(N_1^{(p)} - N_2^{(p)}) \\
&\quad + \frac{i}{2} \Omega_{p,1} e^{i\omega_p t} P_{1}^{(p)} - \frac{1}{2} \Omega_{p,2} e^{-i\omega_p t}, \\
\dot{P}_{1}^{(p)} &= -i[\omega_x - i \Gamma_{p}^{(p)}(N_1^{(p)} - N_2^{(p)})] P_{1}^{(p)} \\
&\quad - \Omega_{p,2} e^{-i\omega_p t} (N_0^{(p)} - N_2^{(p)}), \\
\dot{P}_{2}^{(p)} &= -i[\omega_x - i \Gamma_{p}^{(p)}(N_1^{(p)} - N_2^{(p)})] P_{2}^{(p)} \\
&\quad - \Omega_{p,2} e^{i\omega_p t} (N_0^{(p)} - N_2^{(p)}),
\end{align*}
\]

where the superscript \((p)\) stands for “pump.” We rewrite the polarizations \(P_{1}^{(p)}\) and \(P_{2}^{(p)}\) as the product of a fast oscillatory function and slowly varying variables as

\[
\begin{align*}
P_{1,2}^{(p)} &= \tilde{P}_{1,2}^{(p)} e^{-i\omega_p t}, \\
P_{1,2}^{(p)} &= \tilde{P}_{1,2}^{(p)} e^{i\omega_p t},
\end{align*}
\]

and set the time derivatives of the slowly varying variables \(\tilde{P}_{1,2}^{(p)}\) and \(\tilde{P}_{1,2}^{(p)}\), as well as those of the effective populations \(N_1^{(p)}\), \(N_2^{(p)}\), \(N_0^{(p)}\), and \(N_2^{(p)}\), as zero. In the case of a linearly polarized pump, the magnitudes of pump Rabi frequencies \(\Omega_{p,2}\) and \(\Omega_{p,1}\) are the same:

\[
|\Omega_{p,2}| = |\Omega_{p,1}| = |\Omega_{p}| = \frac{\epsilon^2 |\gamma_h|^2 |E_p|^2}{(2\hbar)^2} \frac{1}{2} \left| |\eta|^2 - \frac{2\kappa \eta}{\sqrt{3}} \cos 2\theta_p \right|.
\]

The population \(N_1^{(p)}\) is the same as \(N_1^{(p)}\), while the population \(N_1^{(p)}\) is identical to \(N_1^{(p)}\). We define the total population of the excited states in presence of the pump and the total population in equilibrium as \(N_0^{(p)}\) and \(N_0^{(0)}\), respectively, as follows:

\[
\begin{align*}
N_1^{(p)} &= N_1^{(0)} = \frac{N_0^{(p)}}{2}, \\
N_2^{(p)} &= N_2^{(0)} = 1 - N_0^{(p)}, \\
N_1^{(p)} &= N_1^{(0)} = \frac{N_0^{(0)}}{2}.
\end{align*}
\]

Although the dephasing rate may be a complicated function of the various populations in the system, for simplicity, the dephasing rate related to EID is modeled as a linear relation with the population differences \(N_1 - N_2\) and \(N_1 - N_1^{(0)}\) as follows:

\[
\Gamma_{2}^{(0)}(N_1 - N_2, N_1 - N_1^{(0)}) = \Gamma_2^{(0)} - \gamma_2(N_1 - N_2) + (N_1 - N_1^{(0)})
\]

\[
- (N_0^{(0)} - N_1^{(0)} - (N_1^{(0)} - N_1^{(0)})) = \frac{\gamma}{2} (N_0^{(p)} - N_0^{(0)}) = -2|\Omega_{p}| (N_0^{(p)} - 1)(\Gamma_2^{(0)} + 2\gamma_2(N_0^{(p)} - N_0^{(0)}))
\]

\[
(\omega_p - \omega_{ex} + i\Gamma_2^{(p)})^2 + [\Gamma_2^{(0)} + 2\gamma_2(N_0^{(p)} - N_0^{(0)})]^2.
\]

We define the effective dephasing constant \(\Gamma_2^{(p)}\) in the presence of the pump as

\[
\Gamma_2^{(p)} = \Gamma_2^{(0)} + 2\gamma_2(N_0^{(p)} - N_0^{(0)}).
\]

The slowly varying variables \(\tilde{P}_{1,2}^{(p)}\) and \(\tilde{P}_{1,1}^{(p)}\) then can be written as

\[
\begin{align*}
\tilde{P}_{1,2}^{(p)} &= \frac{(N_0^{(p)} - 1)\Omega_{p,2}}{\omega_p - \omega_{ex} + i\Gamma_2^{(p)}} = H(\omega_p)\Omega_{p,2}, \\
\tilde{P}_{1,1}^{(p)} &= \frac{(N_0^{(p)} - 1)\Omega_{p,1}}{\omega_p - \omega_{ex} + i\Gamma_2^{(p)}} = H(\omega_p)\Omega_{p,1},
\end{align*}
\]

where the function \(H(\omega_p)\) is defined as

\[
H(\omega_p) = \frac{(N_0^{(p)} - 1)}{\omega_p - \omega_{ex} + i\Gamma_2^{(p)}}.
\]

With the bias solution, we can construct the dynamics equations corresponding to the signal polarizations:
where $P_{11} = P_{12}^{(s)}$, $P_{12} = P_{12}^{(p)}$, and $P_{12} = P_{12}^{(r)}$ are the corresponding polarizations induced by the signal. The details of the derivation will be presented in Appendix A. We denote the frequency difference between the signal and pump as $\delta = \omega_s - \omega_p$. For convenience, we define the functions $D(\omega_s, \omega_p)$, $L(\omega_s, \omega_p)$, $F_i(\omega_s, \omega_p)$, and $F_{h}(\omega_s, \omega_p)$ as

\[
D(\omega_s, \omega_p) = 2 \left( \frac{1}{\omega_s - \omega_p + i\Gamma^{(p)}_2} - \frac{1}{\omega_p - \omega_p - i\Gamma^{(p)}_2} \right),
\]

\[
L(\omega_s, \omega_p) = \frac{1}{\omega_s - \omega_p + i\Gamma^{(p)}_2} - \frac{1}{\omega_p - \delta - \omega_p - i\Gamma^{(p)}_2},
\]

\[
F_i(\omega_s, \omega_p) = \delta + i\gamma_{2,s} - |\Omega|^2 L(\omega_s, \omega_p),
\]

\[
F_{h}(\omega_s, \omega_p) = \delta + i\Gamma_{2,s} - |\Omega|^2 L(\omega_s, \omega_p).
\]

(25)

The relative linear permittivity for TM polarized light then can be written as

\[
\epsilon_r(\omega) = \epsilon_{bgd} + \frac{\Gamma_{||} |\varphi_{c}(\rho_s - \rho_h - 0)|^2}{L_z} \left\{ \frac{2(\lambda^{(p)} - 1)}{\omega_s - \omega_p + i\Gamma^{(p)}_2} \frac{|\Omega|^2}{2\hbar} \left[ 1 + \left( \frac{|\Omega|^2}{2} \right)^2 \right] \right\}
\]

\[
\times \left. \frac{D(\omega_s, \omega_p) \left[ F_i(\omega_s, \omega_p) + F_{h}(\omega_s, \omega_p) + 2|\Omega|^2 L(\omega_s, \omega_p) \right]}{F_i(\omega_s, \omega_p) F_{h}(\omega_s, \omega_p) - |\Omega|^2 L(\omega_s, \omega_p)} \right). \]

(27)

where $\epsilon_{bgd}$ is the relative background permittivity; $\Gamma_{||}$ is the TM confinement factor; $L_z$ is the width of the QW; and $|\varphi_{c}(\rho_s - \rho_h - 0)|^2$ is the square of the absolute values of the excitonic wave function at the origin.\(^\text{17}\)

The permittivity function in Eq. (27) is used to calculate the absorption spectrum, dispersion spectrum, and slowdown factors as a function of signal-pump detuning. Steep variations of the real part of the refractive index are expected in a detuning range characterized by the minimum of the spin-dephasing constants $\gamma_{2,s}$ and $\Gamma_{2,s}$ for V-type EIT. Usually, the coherence between Bloch components of holes decays fast because of the band mixing and fast scattering. Thus, the steep variation will be characterized by the electron-spin-dephasing constant $\gamma_{2,s}$.

For simplicity, we assume that the pump is polarized along the [001] axis. For a TM-polarized signal propagating along the QW plane, we can then write the complex refractive index $n_z(\omega_s)$, the absorption $A_z(\omega_s)$, and the slowdown factor $R_z(\omega_s)$ as

\[
n_z(\omega_s) = \sqrt{\epsilon_{zz}(\omega_s)},
\]

\[
A_z(\omega_s) = 2 \frac{\omega_s}{c} \text{Im}[n_z(\omega_s)],
\]

\[
R_z(\omega_s) = \text{Re}[n_z(\omega_s)] + \omega_c \frac{\partial \text{Re}[n_z(\omega_s)]}{\partial \omega_s}. \tag{28}
\]

3. CALCULATIONS

The system considered contains 75 Å [110] multiple QWs (MQWs). The QW and barrier regions are composed of GaAs and Al$_{0.33}$Ga$_{0.67}$As, respectively. We will focus on the case of uniaxial stress applied along the growth direction. The case of biaxial strain due to lattice mismatch is similar. Some physical properties may change as the strain is changed, e.g., the spin-flip constants, $\Gamma_s$ and $\gamma_s$, spin-dephasing constants $\Gamma_{2,s}$ and $\gamma_{2,s}$, as well as the spontaneous emission constants $\gamma_{1,s}$, $\gamma_{2,s}$, and $\gamma_s$. However, we expect that the order of magnitude for these physical quantities does not significantly change as the uniaxial stress is applied.

We assume that the intrinsic HWHM linewidth $\Gamma^{(0)}_{2,s}$ of the exciton is 2.64 meV. A linewidth of a few millielectron-volts can be easily achieved for Al$_{x}$Ga$_{1-x}$As/GaAs [001] QWs.\(^\text{18}\) We expect that a similar linewidth can be achieved for the ground-state exciton of Al$_{x}$Ga$_{1-x}$As/GaAs [110] QWs. As the uniaxial stress is applied, the linewidth of exciton peaks can change but does not significantly deviate from that of the unstrained case.\(^\text{19}\) The spin-flip constants $\gamma_{s}$ and $\Gamma_{s}$, in the conduction and valence bands are 1.25 ns$^{-1}$ and 0.2 ps$^{-1}$, while the spin-dephasing constants $\gamma_{2,s}$ and $\Gamma_{2,s}$, in the conduction and valence bands are 2.5 ns$^{-1}$ and 0.4 ps$^{-1}$, respectively. Electron-spin coherence is robust against most momentum-scattering processes. In this regard, the spin coherence is not lost when excitons scattered out of and then back into the $k_{CM}$=0 region, where the subscript CM means the center of mass of the exciton. The above assumption is further supported by the recent experimental demonstration of sharp coherent Raman or Zeeman resonance from electron-spin coherences in GaAs QWs.\(^\text{20}\) The magnitude of the spin-dephasing time of the conduction band is set based on the experimental data of a similar scheme for [001] QWs at a low temperature.\(^\text{21}\) Also, for [001] QWs, the much larger hole spin-flip and spin-dephasing constant than those of electrons are due to the fast scattering and band mixing of hole components, and we expect the same is true for [110] QWs. The in-plane spontaneous emission constant $\gamma_{s}$ is set to 2 ns$^{-1}$, and the $z$-polarized spontaneous constant $\gamma_z$ is obtained from the in-plane one together with overlap integrals $\kappa(\Phi \rho | \Psi)$ and $\sigma(\Phi \rho | \Phi)$, so that the ratio of the two spontaneous emission constants is proportional to the square ratio of the absolute values of respective dipole moments. These radiative lifetimes are set to a few hundred picoseconds.\(^\text{22}\) The long radiative lifetime assumed in this model may seem puzzling at first glance since it is well known that the intrinsic radiative lifetime of excitons in high-quality GaAs QWs is of the order of 20 ps. The long radiative lifetime we used is the lifetime.
of a thermal or quasi-thermal exciton distribution, for which only a small fraction of the excitons remain in the region near \( k_{CM}=0 \), where exciton radiative recombination can take place. The EID parameter \( \gamma_2 \) is set to 1.8 ps\(^{-1} \). The magnitude square of the excitonic wave function at the origin \( |\varphi_{ex}(r-p_0=0)|^2 \) is set to \( 3.179 \times 10^{-5} \text{Å}^{-2} \). The TM confinement factor \( \Gamma_z \) of the signal due to finite overlap of the optical field with MQWs is set to 0.2; and the TE pump has intensities set to 0.1, 1, 10, and 100 kW/cm\(^2\).

Figure 3 shows the fraction of LH component \( \int f_{z_0} |\Phi(z_0)|^2 dz_0 \) of the band-edge state as a function of the uniaxial stress. As the uniaxial stress is increased, the top valence band becomes LH-like. The transition is a sharp one and takes place at a stress at \( \sim 2.7 \text{kbar} \). Beyond 2.7 kbar, the top valence band becomes LH-like. We will set the uniaxial stress to 4 kbar in later calculations. Also, note that the coupling between the HH and LH components not only comes from the original \( J=3/2 \) LH Hamiltonian but also from the Bir–Pikus strain Hamiltonian, which is evident from Eq. (1). Thus, the effect of elliptical polarizations comes from both the intrinsic band mixing and the uniaxial stress. This is in contrast to [001] QWs, for which the uniaxial stress only plays the role of shifting the band-edge energy, but not mixing different components. There are no modifications of selection rules for [001] QWs under uniaxial stress.

If an intense TE pump is incident from the top, the background absorption is first saturated, as shown in Fig. 4, for signal absorption spectra under different pump intensities. As the pump intensity increases, an absorption dip emerges in the TM-polarized absorption spectra. The absorption spectra in Fig. 4 are due to V-type EIT. In contrast to population oscillation, which can be induced by the TE-polarized signal, the TM \((110)\)-polarized signal does not cause the real population beating, but still experiences an absorption dip due to quantum interference. The linewidth of the absorption dip for V-type EIT is determined by the electron-spin-dephasing constant \( \gamma_s \) and pump intensity, analogous to that of PO, which is determined by the spontaneous emission constant \( \gamma = \gamma_s + \gamma_e \) and the pump intensity when the TE-polarized signal is used. In the current calculation, the linewidth of the absorption dip for V-type EIT is in the range of several hundr

---

**Fig. 3.** Fraction of the LH component \( \int f_{z_0} |\Phi(z_0)|^2 dz_0 \) of the top valence band edge as a function of the uniaxial stress. As the uniaxial stress is increased, the top valence band gradually becomes LH-like.

**Fig. 4.** Absorption spectra for the TM-polarized signal under different pump intensities. The width of the \( z \)-polarized absorption dip is determined by the long spin lifetime and the TE-polarized pump intensity. The corresponding depth is determined by the saturated background absorption and TE-polarized pump intensity.

**Fig. 5.** Real part of the refractive index for the TM-polarized signal under different pump intensities. The narrower and deeper absorption dips in Fig. 4 correspond to the steeper and larger variations of the real part of the refractive index spectrum.
Lighting can influence the saturation of the background absorption, so that the absorption dip is deep enough to restrict on the optimal intensity. A high pump intensity on the background absorption only puts a loose restriction for the TM-polarized signal. Thus, the saturation characterized by a longer time scale will have a lower slowdown factor. This also accounts for the existence of an optimal intensity because the pump intensity饱和s the background absorption and broadens the absorption dip.

From these slopes, we can calculate the slowdown factors for wave packets with different central frequencies of the $z$-polarized signal, as shown in Fig. 6. The peak slowdown factor does not increase monolithically with the pump intensity because the pump intensity saturates the background absorption and broadens the absorption dip. Ssaturation of the background absorption limits the magnitude of the variation, while the broadening of the dip results in smooth variations. Both effects reduce the positive slope in the real part of the refractive index. For the TM-polarized signal, there is an optimal intensity at which the peak slowdown factor is the highest. Usually, the process characterized by a longer time scale will have a lower optimal intensity and higher slowdown factor. This also accounts for the existence of an optimal intensity because the increase of the pump intensity effectively enhances the spin dephasing rate, which leads to a shorter time scale.

Figure 7 shows the peak slowdown factor as a function of the pump intensity. Because of a large TM-polarized dipole moment and a small TE-polarized one, the background absorption is not easily saturated by the TE-polarized pump, but can provide a significant dipole interaction for the TM-polarized signal. Thus, the saturation of the background absorption only puts a loose restriction on the optimal intensity. A high pump intensity is required, so that the absorption dip is deep enough to be influenced by the saturation of the background absorp-

tion. The drop of the peak slowdown factor above the optimal intensity is mainly limited by the pump-induced broadening of the absorption dip. From Fig. 4, the linewidth of the absorption dip is significantly broadened before the percentage of the absorption dip is about one half of the background absorption (the curve corresponding to a pump intensity of 100 kW/cm²).

A current limitation to the slowdown factor is the low confinement factor, because only the waveguide geometry can be used for the TM-polarized signal. The slowdown factor can be more prominent if the overlap between the optical field and the QW region can be improved, e.g., by using multiple quantum wells. On the other hand, increasing the confinement factor will also reduce the transmission power intensity due to the residual absorption. The absorption also limits the time delay because the device cannot be too long. A fractional bit delay of $\sim 0.5$ is estimated for an allowable 20 dB signal attenuation. The optimal performance should be investigated for a large slowdown factor with a measurable transmission power. On the other hand, the possibility of the integrated optical circuit in semiconductors makes it possible to cascade several stages composed of an optical buffer of this type and a broadband gain medium in series. Therefore, a significant time delay can be accumulated in several stages to compensate for the loss in the optical buffer. In real applications, there are also issues such as high-order dispersions and signal-to-noise ratio due to the successive attenuation and spontaneous emission in the gain medium.23–25 However, they are beyond the scope of this paper and will be left for future work.

4. CONCLUSION

We have considered the slow light using V-type EIT for strained [110] QWs. The long electron-spin lifetime of [110] QWs can enhance the slowdown factor for TM-polarized light. The uniaxial stress and tensile biaxial strain can make the LH-like band the top valence band and reduce unwanted absorption and relaxation from the HH-like continuum. Unlike [001] QWs, the selection rules of excitonic transitions have to be reformulated because of the mixing from LK and Bir–Pikus Hamiltonians. A slowdown factor of a few thousand can be achieved using V-type EIT. Further enhancement of the slowdown factor is possible if the confinement factor of the signal can be improved.

APPENDIX: V-TYPE ELECTROMAGNETICALLY INDUCED TRANSPARENCY FOR TRANSVERSE-MAGNETIC POLARIZED LIGHT

We use perturbation theory to write the dynamics equations of the polarizations $P_{11}^{(1)}$, $P_{11}^{(2)}$, $P_{11}^{(s)}$, and $P_{12}^{(s)}$. After keeping the linear terms, the dynamics equations of the signal polarizations are
The above signal polarizations are rewritten in terms of the slowly varying variables \( \tilde{P}^{(s)}_{1}(\delta), \tilde{P}^{(s)}_{1}(\delta), \tilde{P}^{(s)}_{12}(\delta), \) and \( \tilde{P}^{(s)}_{12}(\delta) \) as

\[
\begin{align*}
P^{(s)}_{1} &= \tilde{P}^{(s)}_{1}(\delta)e^{-i\delta t} + \tilde{P}^{(s)}_{1}(\delta)e^{i\delta t}, \\
P^{(s)}_{1} &= \tilde{P}^{(s)}_{1}(\omega_{s})e^{-i\omega_{s}t} + \tilde{P}^{(s)}_{1}(\omega_{p})e^{-i\omega_{p}t}, \\
P^{(s)}_{1} &= \tilde{P}^{(s)}_{12}(\omega_{s})e^{-i\omega_{s}t} + \tilde{P}^{(s)}_{12}(\omega_{p})e^{-i\omega_{p}t}, \\
P^{(s)}_{12} &= \tilde{P}^{(s)}_{12}(\omega_{s})e^{-i\omega_{s}t} + \tilde{P}^{(s)}_{12}(\omega_{p})e^{-i\omega_{p}t}.
\end{align*}
\]

(A2)

Substituting Eq. (A2) into Eq. (A1), neglecting the time derivatives of the slowly varying variables, and comparing the frequency components at both sides of the dynamics equations, we find that the slowly varying variables of the signal polarizations are decoupled into two groups: \( \tilde{P}^{(s)}_{1}(\delta), \tilde{P}^{(s)}_{1}(\omega_{s}), \) \( \tilde{P}^{(s)}_{1}(\omega_{p}), \) \( \tilde{P}^{(s)}_{12}(\delta), \) and \( \tilde{P}^{(s)}_{12}(\omega_{s}), \) \( \tilde{P}^{(s)}_{12}(\omega_{p}), \) \( \tilde{P}^{(s)}_{12}(\omega_{s}), \) and \( \tilde{P}^{(s)}_{12}(\delta), \) form one group, while \( \tilde{P}^{(s)}_{1}(\omega_{p}), \tilde{P}^{(s)}_{1}(\omega_{s}), \) \( \tilde{P}^{(s)}_{12}(\omega_{s}), \) \( \tilde{P}^{(s)}_{12}(\omega_{p}), \) \( \tilde{P}^{(s)}_{12}(\omega_{s}), \) and \( \tilde{P}^{(s)}_{12}(\delta), \) form the other.

The slowly varying variables \( \tilde{P}^{(s)}_{1}(\omega_{s}) \) and \( \tilde{P}^{(s)}_{1}(\omega_{p}) \) can be written in terms of the variables \( \tilde{P}^{(s)}_{1}(\delta) \) and \( \tilde{P}^{(s)}_{12}(\delta) \) as

\[
\begin{align*}
\tilde{P}^{(s)}_{1}(\omega_{s}) &= \frac{\Omega_{p,1}}{\omega_{s} - \omega_{ex} + i\Gamma_{p}^{2}} \tilde{P}^{(s)}_{1}(\delta) - \frac{\Omega_{p,2}}{\omega_{s} - \omega_{ex} + i\Gamma_{p}^{2}} \tilde{P}^{(s)}_{12}(\delta) \\
&\quad \quad + \frac{N^{(p)} - 1}{\omega_{s} - \omega_{ex} + i\Gamma_{p}^{2} \Omega_{s,11}}, \\
\tilde{P}^{(s)}_{1}(\omega_{p}) &= \frac{\Omega_{p,2}}{\omega_{p} - \omega_{ex} + i\Gamma_{p}^{2}} \tilde{P}^{(s)}_{1}(\delta) \\
&\quad \quad - \frac{\Omega_{p,1}}{\omega_{p} - \omega_{ex} + i\Gamma_{p}^{2}} \tilde{P}^{(s)}_{12}(\delta).
\end{align*}
\]

(A3)

Similarly, the slowly varying variables \( \tilde{P}^{(s)}_{1}(\omega_{s}) \) and \( \tilde{P}^{(s)}_{12}(\omega_{p} - \delta) \) are written in terms of the variables \( \tilde{P}^{(s)}_{1}(\delta) \) and \( \tilde{P}^{(s)}_{12}(\delta) \) as

\[
\begin{align*}
\tilde{P}^{(s)}_{1}(\omega_{s})(\omega_{s}) &= \frac{\Omega_{p,1}}{\omega_{s} - \omega_{ex}} + i\Gamma_{p}^{2} \tilde{P}^{(s)}_{1}(\delta) - \frac{\Omega_{p,2}}{\omega_{s} - \omega_{ex}} + i\Gamma_{p}^{2} \tilde{P}^{(s)}_{12}(\delta) \\
&\quad \quad + \frac{N^{(p)} - 1}{\omega_{s} - \omega_{ex} + i\Gamma_{p}^{2} \Omega_{s,11}}, \\
\tilde{P}^{(s)}_{12}(\omega_{p} - \delta) &= \frac{\Omega_{p,2}}{\omega_{p} - \omega_{ex}} \tilde{P}^{(s)}_{1}(\delta) \\
&\quad \quad - \frac{\Omega_{p,1}}{\omega_{p} - \omega_{ex}} \tilde{P}^{(s)}_{12}(\delta).
\end{align*}
\]

We substitute Eq. (A6) into Eqs. (A3) and (A4) and rewrite the expressions of the slowly varying variables \( \tilde{P}^{(s)}_{1}(\omega_{s}) \) and \( \tilde{P}^{(s)}_{12}(\omega_{p} - \delta) \) as follows:
Using Eq. (A7), we can obtain the linear polarization density $P^L_p$ and the displacement density $D^L_s$ caused by V-type EIT as

$$P^L_p = \frac{\Gamma_e}{L_z} \left[ (\phi_e | \varepsilon_s | \varphi_h)^2 + (\varphi_h | \phi_e | \varepsilon_s)^2 \right] e^{i\omega_p t},$$

$$D^L_s = \frac{\varepsilon_0 \varepsilon_{sd}}{2} E_{s,s} e^{-i\omega_s t} - P^L_p = \frac{\varepsilon_0 \varepsilon_s(\omega)}{2} - \frac{e_p}{2}. \quad (A8)$$

Substituting Eqs. (6) and (A7) into Eq. (A8), we can then obtain the expression of the relative linear permittivity $\varepsilon_s(\omega)$ in Eq. (27).

ACKNOWLEDGMENTS

This work was supported by Defense Advanced Research Projects Agency under grants AFSA3631-22549 (University of Illinois at Urbana-Champaign) and F30602-02-2-0096 (University of California at Berkeley), joint contract under U.S. Air Force Office of Scientific Research FA 9550-04-1-0196, and the Army Research Office.

S. L. Chuang's e-mail address is s-chuang@uiuc.edu.

REFERENCES


